Cross Sections of Three-Dimensional Figures

Getting the Idea

A three-dimensional figure (also called a solid figure) has length, width, and height. It is not flat. Some examples of three-dimensional figures are below.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A prism</strong></td>
<td>Has a pair of bases that are parallel, congruent polygons. Its other faces are rectangles.</td>
</tr>
<tr>
<td><strong>A rectangular prism</strong></td>
<td>Has 6 faces that are rectangles.</td>
</tr>
<tr>
<td><strong>A cube</strong></td>
<td>Is a rectangular prism with 6 square faces.</td>
</tr>
<tr>
<td><strong>A pyramid</strong></td>
<td>Has one base that is a polygon. Its other faces are triangles. The height of a pyramid is called its altitude, and the height of its lateral face is called its slant height.</td>
</tr>
<tr>
<td><strong>A rectangular pyramid</strong></td>
<td>Has a base that is a rectangle.</td>
</tr>
<tr>
<td><strong>A square pyramid</strong></td>
<td>Has a base that is a square.</td>
</tr>
</tbody>
</table>

A three-dimensional figure can be sliced by a plane to show a two dimensional view. This view is called a cross section.

Example 1

A square pyramid is sliced by a plane that is parallel to its base, as shown.

What is the shape of the cross section?

**Strategy**

Visualize a plane, parallel to the base, slicing through the pyramid. The cross section will have the same shape as the base. It will be a square.

**Solution**

The shape of the cross section is a square.
Using the clay:

1. Construct a cube
   a) slice horizontally from the base. What shape is it?
   b) slice vertically to the base. What shape is it?

2) Construct a rectangular prism.
   a) slice horizontally from the base. What shape is it?
   b) slice vertically to the base. What shape is it?

3) Construct a rectangular pyramid.
   a) slice horizontally from the base. What shape is it?
   b) slice vertically from the base through the vertex. What shape is it?
   c) slice vertically to the base not through the vertex. What shape is it?

Conjectures:
Example 2
A rectangular prism is cut by the slanted plane shown.
What is the shape of the cross section?

Strategy
Visualize the prism being sliced by a thin piece of wire.

Determine the angle at which the plane intersects the prism.
The plane is neither horizontal nor vertical to the faces of the prism.

Imagine slicing the rectangular prism with a piece of wire.

The prism is now in two parts.

Visualize the shape of the cross section.

Solution
The shape of the cross section is a parallelogram.

Example 3
Look at this cube.
How can a plane slice the cube so that the cross section is a triangle?

Strategy
Visualize using a plane to slice the cube to get a triangular cross section.
Slice through a corner of the cube with a plane.

Solution  The cross section of the cube is shown above.

**Coached Example**

Nari will slice this pyramid with a plane that is perpendicular to the base and passes through the top vertex.

What is the shape of the cross section?

What does “perpendicular” mean?

Visualize slicing the prism with a plane that is perpendicular to the base and passes through the top vertex.

Make a sketch of the cross section in the space below.

The shape of the cross section is ________________.
Choose the correct answer.

1. The rectangular prism is being sliced by a plane parallel to its base.

   What will be the shape of the cross section formed?
   A. parallelogram that is not a rectangle
   B. rectangle that is not a square
   C. square
   D. triangle

2. What is the shape of the cross section formed when the square pyramid is sliced by a plane perpendicular to its base that does not pass through its top vertex?

   A. parallelogram (not a square)
   B. square
   C. trapezoid
   D. triangle

3. What is the shape of the cross section formed when the rectangular pyramid is cut by the plane parallel to its base?

   A.
   B.
   C.
   D.
4. Matt molded a cube from clay. He then took piano wire and sliced through the clay, as shown by the dotted lines below.

What is the shape of the cross section formed when he does this?
A. rectangle (not a square)
B. square
C. trapezoid
D. triangle

5. Which is the shape of the cross section formed when the triangular pyramid is cut by the plane, as shown?

A.  
B.  
C.  
D.  

6. The diagram shows a cube with each edge 2 feet long. The cube has been cut by a plane perpendicular to its base.

A. Identify and draw the shape of the cross section formed when the cube was cut by the plane shown. Label its dimensions.

B. Suppose the plane were parallel to the base of the cube. What would be the shape of the cross section now? Explain your answer.
7. Draw a line from each cube to its cross section.

A.

B.

C.

8. A square prism is sliced by a plane that is perpendicular to its base. Circle the cross section.

The cross section is a

- triangle
- square
- rectangle
- trapezoid
9. A rectangular pyramid is sliced by a plane. Select True or False for each statement.

![Image of a pyramid with a slice]

A. If the slice is parallel to the pyramid's base, the cross section is a triangle. ○ True ○ False

B. If the slice is perpendicular to the pyramid's base but not through the vertex, the cross section is a triangle. ○ True ○ False

C. If the slice is perpendicular to the pyramid's base through the vertex, the cross section is a triangle. ○ True ○ False

D. If the slice is neither parallel nor perpendicular to the pyramid's base, the cross section is a trapezoid. ○ True ○ False

10. Look at each figure. Is the cross section a triangle? Select Yes or No.

A. ![Image of a pyramid slice]
   ○ Yes ○ No

B. ![Image of a pyramid slice]
   ○ Yes ○ No

C. ![Image of a pyramid slice]
   ○ Yes ○ No

D. ![Image of a pyramid slice]
   ○ Yes ○ No